Comment on "Self-interacting Warm Dark Matter"

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Hannestad and Scherrer analysed self-interacting Warm and Hot Dark Matter and found power spectra in disagreement with previous papers. We argue that they include self-interactions in a weak-coupling approximation, as for photons after recombination. The "tightly coupled" approximation used for pre-recombination photons, and previous discussions of self-interacting HMD, would have been more appropriate. Their approximation generates a Boltzmann hierarchy leading to a stiff system of equations. Furthermore contamination by gauge modes could invalidate their results.

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Neutrinos with strong self-interactions were originally suggested as a dark matter candidate* by Raffelt and Silk [1], who estimated that density perturbations in such a fluid would damp by diffusion ("Silk damping"), rather than by free-streaming. In a subsequent analysis—partially numerical, partially analytic—Atrio-Barandela and Davidson (ABD) [2] computed a power spectrum in agreement with this expectation. More recently, power spectra were computed by Hannestad and Scherrer [3] for self-interacting hot and warm dark matter, using CMBFAST [4]. They find that the shape of the power spectrum is essentially independent of the strength of the dark matter self-interactions, but depends on the mass of the dark matter particle, suggesting that the damping of density fluctuations is the due to free-streaming rather than diffusion. Their power spectra do not show the damping due to diffusion and the short distance oscillations that one would expect for an interacting dark matter fluid [1,2]. We believe this is due to an inappropriate analytic approximation, and possibly to contamination by gauge modes.

HS study the evolution of perturbations $\Psi(x^i, q_i, \tau)$ in the dark matter phase space distribution f:

$$f(x^i, q_i, \tau) = f_0(1 + \Psi(x^i, q_i, \tau)) \tag{1}$$

f_0 is the homogeneous and isotropic equilibrium phase space distribution, and x^i, τ are comoving coordinates (see [5] for notation). In the absence of interactions, f satisfies the Boltzman equation for free particles

$$\mathcal{L}[f] = \frac{\partial f}{\partial \tau} + \frac{\partial x^i}{\partial \tau}\frac{\partial f}{\partial x^i} + \frac{\partial q_j}{\partial \tau}\frac{\partial f}{\partial q_j} = 0 \tag{2}$$

The CMBFAST Boltzman code solves this equation for Ψ to evolve the HDM density perturbations.

The Boltzman equation for an interacting species is

$$\mathcal{L}[f] = \mathcal{C}[f] \tag{3}$$

where the collision term $\mathcal{C}[f]$ is difficult to solve. Schematically:

$$\mathcal{C}[f_a] = \frac{1}{2E_a}\int d\Pi_b d\Pi_c d\Pi_d |\mathcal{M}|^2 \hat{\delta}(f_a f_b - f_c f_d) \tag{4}$$

where $d\Pi_b = d^3p_b/[(2\pi)^3 2E_b]$, $\mathcal{M}$ is the matrix element for the process $a + b \leftrightarrow c + d$, $\hat{\delta}$ is the 4-momentum conservation delta-function, and we have assumed low occupation numbers. An *equilibrium* distribution is defined to satisfy $\mathcal{C}[f] = 0$, so $\mathcal{L}[f_0] = 0 = \mathcal{C}[f_0]$, and

$$\mathcal{L}[f_0\Psi] = \mathcal{C}[f_0\Psi] \tag{5}$$

as noted by HS. We first discuss an approximate solution suitable for strongly interacting particles, and then compare to the method of HS.

The timescale associated with the LHS of equation (5), for horizon-sized modes, is the age of the Universe τ_U. The timescale associated with the RHS is the interaction timescale τ_ν and $\tau_\nu \ll \tau_U$ for strong self-interactions. In this case, (5) can be solved in the "tightly-coupled" approximation used for pre-recombination photons: perturbations will be very close to locally in equilibrium, because the interactions are fast. So

$$f(x^i, q_i, \tau) = g^{(0)}(x^i, \tau) + g^{(1)}(x^i, q_i, \tau) \tag{6}$$

where

$$g^{(0)}(x^i, \tau) = \exp\{(E - \mu(x^i, \tau))/T(x^i, \tau)\} \quad , \tag{7}$$

is the locally in equilibrium distribution, so $g^{(1)}$ is the perturbation away from *local* equilibrium. By definition $\mathcal{C}[g^{(0)}] = 0$, so the Boltzman equation is

$$\mathcal{L}[f_0\Psi] = \mathcal{C}[g^{(1)}] \simeq \Gamma g^{(1)} \quad (\Gamma = \tau_\nu^{-1}) \tag{8}$$

This implies $g^{(1)} \sim (\tau_\nu/\tau_U) f_0 \Psi$. The equality in eqn (8) is the analytic approximation required to generate a hierarchy of Boltzmann equations similar to the one

*a light bosonic particle could behave very differently [7].

solved by HS. From the approximation in eqn (8), we showed that density perturbations in interacting HDM models would be damped by diffusion [2], as anticipated in [1]. (This is also the result obtained by HS in their Appendix 2.)

HS used a different approach from us; they take

$$\mathcal{C}[\Psi] \simeq \Gamma\Psi \tag{9}$$

and generate a Boltzmann hierarchy. The resulting equations are, however, very stiff. Their approximation (equation 9) diverges in the limit of very large scattering cross sections and will not model the fact that fast interactions can bring Ψ into a local equilibrium form almost satisfying $\mathcal{C}[\Psi] = 0$. Therefore, we believe that equation (9) does not correctly model the HDM self-interactions, and this could explain why HS's power spectra look like non-interacting Hot Dark Matter. Equation (9) would be valid in the weak coupling approximation used for photons after recombination, as opposed to the pre-recombination tight coupling approximation leading to equation (8).

Note that both ABD and HS work in synchronous gauge, where density perturbations in the CDM regime are defined in the matter rest frame. However, the divergence of the fluid velocity θ is non-zero for relativistic neutrinos, so one must verify that $\theta_\nu = 0$ after the ν become non-relativistic, to ensure that unphysical gauge modes do not contaminate the solution. This could be problematic in the full numerical approach of HS.

As mentioned, HS discuss the relation of their paper to ABD in an Appendix, where they reproduce the analytic estimates [1,2] showing that the perturbations should oscillate within the horizon and be damped on the diffusion scale. They do not comment on the discrepancy between the estimates and their plots. They suggest that the difference between the ABD $P(k)$ plot and theirs is due to an extra $H\theta$ term appearing in our equations, where θ is the divergence of the fluid velocity. This term is due to a difference in conventions: as stated in our paper, we use the conventions of [6] for our analytic derivations, so $\theta = ik \cdot v/a$, and our equations are correct (see eqn 85.8 in [6]). HS use the conventions of [5], according to which $\theta = ik \cdot v$, and the offending $H\theta$ term does not appear. In MB notation, where δ represents the density perturbation, equations ABD [40] and [41] are:

$$\dot{\delta} = -(1+w)(\theta + \frac{1}{2}\dot{h}) - 3\frac{\dot{a}}{a}(\frac{\dot{P}}{\dot{\rho}} - w)\delta \tag{10}$$

$$\dot{\theta} = -\frac{\dot{a}}{a}(1-3w)\theta - \frac{\dot{w}}{1+w}\theta + k^2\frac{\dot{P}/\dot{\rho}}{1+w}\delta - k^2\sigma(\theta - 3\dot{\eta}) \tag{11}$$

where $w = P/\rho$, P = pressure, ρ = density, a is the scale factor, σ the viscosity, k the wavelength in conformal units, h and η are the metric perturbations, and derivatives are with respect to conformal time. In the relativistic regime $w = 1/3, \dot{w} = 0$ and neutrinos behave like a damped harmonic oscillator, while in the non-relativistic regime density perturbations grow like Cold Dark Matter. However, HS are correct to doubt our plots; our treatment of the transition from relativistic neutrino fluid to non-relativistic fluid was simplistic, and we did not correctly take into account the effect of viscosity on small scales (a factor 3 was missing in our programme). During the transition period from radiation to matter dominated regimes we assumed that $w = P/\rho$, was linear with conformal time. Other relationships gave very similar power spectra. We have corrected the viscosity term, and plot the power spectrum in figure 1. This correction has a negligeable effect: the small scale oscillations are slightly reduced with respect to [2], and our results agree with expectations.

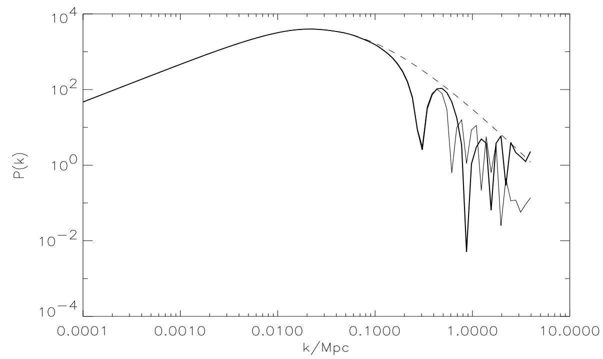

FIG. 1. Power spectrum for two different "sticky neutrino" models. The solid lines correspond (in decreasing amplitude) to interactions with co-moving mean free paths at $T_\gamma = 10$ eV of .01 Mpc and .1 Mpc. These give rise to estimated damping scales of 7 Mpc^{-1} and 3 Mpc^{-1}, respectively. The dashed line correspond to standard CDM and is plotted for comparison. The y-axis scale is arbitrary. We took $H_o = 50$Km s^{-1}Mpc^{-1}.

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